

Popular Computing

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The Four 4's

The Four Fours problem appeared in the Graham Dial magazine in November, 1943. The problem stated then was to represent the number 71 with exactly four 4's, with the simplest solution given as

$$\frac{4! + 4.4}{.4} = 71$$

The problem is reprinted as No. 17 in the book Ingenious Mathematical Problems and Methods, by L. A. Graham, Dover Publications, 1959.

A great many integers can be so represented, ranging from zero (44 - 44) to fantastic heights. The number given by

$$4^{4^{4^4}} \quad (A)$$

is of the order of 2^Q where $Q = 2.681561585988518 \times 10^{154}$.
For the number

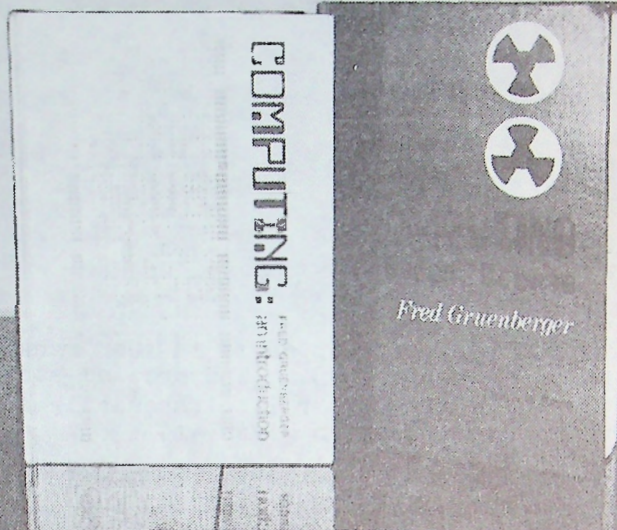
$$4!^{4!^{4!^{4!}}} \quad (B)$$

it would be difficult even to conjecture as to the order of magnitude of the result.

Please turn to page 3

COMPUTING: AN INTRODUCTION, Harcourt Brace Jovanovich, 1969
757 Third Avenue, New York City 10017

COMPUTING: A SECOND COURSE, Canfield Press, 1971
850 Montgomery Street, San Francisco 94133



"...still the only integrated set of
introductory texts in the field."

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Among small integers, solutions to the four 4's problem are readily obtained up to 154. Some of the more ingenious ones are:

$$4! - 4 - \frac{4}{4} = 19$$

$$\frac{44 - \sqrt{4}}{\sqrt{4}} = 21$$

$$\frac{\frac{4!}{.4} + \sqrt{4}}{\sqrt{4}} = 31$$

$$44 + 4 + !4 = 57$$

(where $!4 = 9$, the subfactorial function, tabulated in PC-1).

A list of solutions to the four 4's problem, for the numbers from 1 to 120, will be sent on receipt of a stamped, self-addressed envelope.

Solutions are not known for the following numbers: 155, 157, 158, 161, 165, 166, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, and many integers greater than 200.

The ability to represent extremely large numbers with a few digits, as in (A) above, was highlighted in discussions in Mathematical Tables and other Aids to Computation in 1946. In short articles in issues 14 and 17, R. C. Archibald and Horace Uhler indicated what was then known about the quantity

$$99^9$$

(C)

Uhler gave the 138 high order digits of this number; Archibald gave the 26 low order digits. The number is

$9^{387420489}$ and contains 369693100 digits.

The accompanying printouts give the low order 2000 digits (probably all correct) and the high order 700 digits, of which the last 7 digits are known to be incorrect.

809525958589922508866655754287362207938630669145320812550249
 850876231167399049720427454018012746386257248980350758867746
 351340999939041634521726538101260454013633427664136780855764
 637995217610229048057174542389836334625373985688612504472489
 490452854812034007825529932532252730519950474474028709929662
 551829505557524153564665719954033989006618620089926249234540
 736596776221983170581823427603175337026319250915305776294828
 846915778150179743572322770911274875206868938870681955008307
 722567099206594850660393232184955152285531403439472970290714
 735729172978497892788643154958968536830282720050835996766928
 960029914262248863062289601851525320099966009604253619228138
 670235968612352604841875817838539716952049111295367896277149
 357959887563972803574391654775437573514446532564393005262061
 966198400192799471949506855899634491822546579930695531048952
 608483866895848857944161209096255511247427978972881431604590
 674367091084214694838099301244442639237145620050167498640435
 621625000302386605281834511393319407267916437894757477844731
 142780767037267003263590250822342923877464623911279751529028
 988405927046285969119001418842032015433264756391857718597649
 204912230323811027210136918368494328574137376263796912584561
 412374440601020026085922354106227707187022304023593564191512
 969962866684600663029835137902721579657456534443278490334199
 454357557541697596627896410612703879902561283536679505899361
 171724902858145717339151876022832813835586657889953502722539
 543451659839173364275071543317493863779576502233071689586371
 971921105787378573369432124577155212755139983177854767167859
 129964506729627483736530221523432050747834092790565371273832
 640535909769963513435977537992836807528175483827244781445369
 409799723047184176258944795154018072624283659761429188348967
 918815377285476781074966161266185476266685323552900557188849
 167988554700684735826850897391870085107540281885392534905291
 22882039719724032235770060732838773582826170043150602250406
 601961656994397543610268552663740366829061901749234943241787
 99359681422627177289

The low-order 2000 digits of 99^9

042812477317574704803698711593056352133905548224144351417475
 372305352388747173504835319366529943203337506041753364763100
 078032613904733860832080206037470612809165574132086446019861
 999614520310524428581489598115147194935176779655930215939338
 501502396942623105296758165694333346314749255384948593377812
 087624957216504195220601804571301517864051014594079041948663
 327336671830654410760234823633427933884734491714907139283876
 367034707332216158426388470264465058580355824823115778277866
 180114720994362906904734383664886646950233817353314932888115
 176124859712015335756443987605995621733954850395053696554453
 295547762183338179903753742986603617541076696090471833992393
 31453425470226983330651282587035206474904

The high-order 700 digits of 99^9

Desk Calculator Review

The machines reviewed here all have similar logic; they include Craig, Bowmar, Sears 5885, TI-2500, Summit, Regan, Rapidman, and Brother.

These machines, which sell in the \$80-\$120 range, have common characteristics. All of them operate in floating decimal, with an 8-digit display. They have storage for one constant multiplier/divisor. Some have a feature wherein the display goes out after 15 seconds or so of inactivity; the display can be restored on depressing the restore key. All of them permit chained operations; thus the calculation of

$$\frac{3.7 \cdot .00635}{216.74} + 37 = 1.6087003$$

23

can be performed in one sequence without reentering any numbers.

The constant divisor allows for taking reciprocals. The reciprocal of the result shown above may be obtained by the following sequence:

1. K (constant switch) ON.
2. DIVIDE, EQUALS (produces 1.).
3. K OFF.
4. DIVIDE, EQUALS (produces .6216198).

Since automatic squaring is available on these machines, the following sequence is interesting. Form $4097/4096 = 1.0002441$. Square (that is, press TIMES, EQUALS) 12 times, to produce 2.7169548. This is equivalent to

$$\left(1 + \frac{1}{2^{12}}\right)^{2^{12}} \sim e$$

The following sequence can be used to demonstrate the division speed:

1. K ON.
2. Enter .999.
3. DIVIDE.
4. Press EQUALS and repeat step 4.

Each depression of EQUALS yields a full 8-digit quotient, and this operation can be performed rapidly. One hundred depressions of EQUALS produces 1.1052218.

Similarly, the number 1.0000001 can be entered and squared 27 times (yielding 671189.63); the original number has thus been raised to the 134000000th power directly.

For those machines using the Texas Instruments TMS 0100 NC series chip, calculations are performed in milliseconds, the slowest operation being a division, at 35 ms.

Most of these machines operate on rechargeable batteries, allowing for 3-5 hours of use away from AC power; when fully exhausted, the battery pack takes from 10-15 hours to recharge fully.

Provided that the user is content with simple arithmetic (that is, can forego the pleasure of logs and trigonometric functions as on the Hewlett-Packard HP-35), any of these machines is an excellent tool.

The \$100-class machines can also be obtained with disposable batteries as the power source, or with straight AC power. The choice depends on the intended usage. For example, boating enthusiasts who use the machines for navigation would probably prefer disposable batteries (and carry lots of spares). If all work is to be done at a desk, the pure AC operation is not inconvenient, and will save money on the purchase price.

The introduction of the current breed of electronic desk calculators has forced some new notation. We have the following forms of arithmetic:

1. Fixed point. This is epitomized by the action on mechanical adding machines, wherein all entries are taken in the form xxxxxx.xx.

2. Variable fixed point. The less expensive electronic machines provide for varying the position of the fixed decimal point from 1 to 6 places. With the switch set at 2, the action is then the same as the action in the mechanical adding machine.

3. Floating point. Decimal points are aligned automatically. Thus, the calculation shown in the first example above can be performed without the user having to keep track of the decimal positioning. With this feature, the machines tend to produce all results to 8 (for example) significant digits, with leading zeros suppressed to the left of the decimal point in the display, and trailing zeros suppressed after the decimal point.

4. Scientific notation. In this mode of arithmetic, the number 1000 pi can be entered as

3.1415927 E03,

and results will be in the same notation. The range on the exponent is commonly ± 99 . The machines under review here do not offer this feature.



The Way To Learn Computing Is To Compute



SEMPER ERRO; NUMQUAM DUBITO

2

Log 2	0.3010299956639811952137388947244930267681898814621
Ln 2	0.6931471805599453094172321214581765680755001343603
$\sqrt{2}$	1.4142135623730950488016887242096980785696718753769
$\sqrt[3]{2}$	1.2599210498948731647672106072782283505702514647015
$\sqrt[4]{2}$	1.1486983549970350067986269467779275894438508890978
$\sqrt[5]{2}$	1.1040895136738123376495053876233447213253266007801
$\sqrt[10]{2}$	1.0717734625362931642130063250233420229063846049775
$\sqrt[100]{2}$	1.0069555500567188088326982141132397854535407405341
e^2	7.3890560989306502272304274605750078131803155705518
π^2	9.8696044010893586188344909998761511353136994072407
$\tan^{-1} 2$	1.1071487177940905030170654601785370400700476454014
2^{100}	1267650600228229401496703205376.
2^{1000}	107150860718626732094842504906000181056140481170553 360744375038837035105112493612249319837881569585812 759467291755314682518714528569231404359845775746985 748039345677748242309854210746050623711418779541821 530464749835819412673987675591655439460770629145711 96477686542167660429831652624386837205668069376.

FACTORIALS

In the following table, entries up to 28! are exact, after which the first 30 significant digits are given and the number of digits to the decimal point.

1	1	
2	2	
3	6	
4	24	
5	120	
6	720	
7	5040	
8	40320	
9	362880	
10	3628800	
11	39916800	
12	479001600	
13	6227020800	
14	87178291200	
15	1307674368000	
16	20922789888000	
17	355687428096000	
18	6402373705728000	
19	121645100408832000	
20	2432902008176640000	
21	51090942171709440000	
22	1124000727777607680000	
23	25852016738884976640000	
24	620448401733239439360000	
25	15511210043330985984000000	
26	403291461126605635584000000	
27	10888869450418352160768000000	
28	304888344611713860501504000000	
29	884176199373970195454361600000	0001
30	265252859812191058636308480000	0003
40	815915283247897734345611269596	0018
50	304140932017133780436126081660	0035
60	832098711274139014427634118322	0052
70	119785716699698917960727837216	0071
80	715694570462638022948115337231	0089
90	148571596448176149730952273362	0109
100	933262154439441526816992388562	0128

150	571338395644585459047893286526	0233
200	788657867364790503552363213932	0345
250	323285626090910773232081455202	0463
300	306057512216440636035370461297	0585
350	123587405826548875014395199766	0711
400	640345228466238952623479703195	0839
500	122013682599111006870123878542	1105
600	126557231622543074254186782451	1379
700	242204012475027217986787509381	1660
800	771053011335386004144639397775	1947
900	675268022096458415838790613618	2240

402387260077093773543702433923003985719374864210714632543799
 910429938512398629020592044208486969404800479988610197196058
 631666872994808558901323829669944590997424504087073759918823
 62772718873251977950595095276120874975462497043601418278094
 646496291056393887437886487337119181045825783647849977012476
 632889835955735432513185323958463075557409114262417474349347
 553428646576611667797396668820291207379143853719588249808126
 867838374559731746136085379534524221586593201928090878297308
 431392844403281231558611036976801357304216168747609675871348
 312025478589320767169132448426236131412508780208000261683151
 027341827977704784635868170164365024153691398281264810213092
 761244896359928705114964975419909342221566832572080821333186
 11681155361583654698404670897560290095053761647584728421889
 679646244945160765353408198901385442487984959953319101723355
 556602139450399736280750137837615307127761926849034352625200
 015888535147331611702103968175921510907788019393178114194545
 257223865541461062892187960223838971476088506276862967146674
 697562911234082439208160153780889893964518263243671616762179
 168909779911903754031274622289988005195444414282012187361745
 992642956581746628302955570299024324153181617210465832036786
 906117260158783520751516284225540265170483304226143974286933
 06169089796848259012545832716822645806652676995865268272807
 07578139185817888965220816434834825993266043367660176999612
 831860788386150279465955131156552036093988180612138558600301
 435694527224206344631797460594682573103790084024432438465657
 245014402821885252470935190620929023136493273497565513958720
 559654228749774011413346962715422845862377387538230483865688
 97646192738381490014076731044664025989949022221765904339901
 886018566526485061799702356193897017860040811889729918311021
 171229845901641921068884387121855646124960798722908519296819
 372388642614839657382291123125024186649353143970137428531926
 649875337218940694281434118520158014123344828015051399694290
 153483077644569099073152433278288269864602789864321139083506
 217095002597389863554277196742822248757586765752344220207573
 630569498825087968928162753848863396909959826280956121450994
 871701244516461260379029309120889086942028510640182154399457
 156805941872748998094254742173582401063677404595741785160829
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This is factorial one thousand.

Book Review

THE DIGITAL VILLAIN, by Robert M. Baer, Addison-Wesley, 1972, paper, 187 pages, \$2.95.

It is difficult to decide just what audience this book is aimed at. The author states that it has been used in an introductory course at UC Berkeley for some years. It is thus one of those books which, used in a course given by the author, can be excellent. But used by someone else, it could be a disaster.

The book's subtitle is "Notes on the Numerology, Parapsychology, and Metaphysics of the Computer." It is all of that--for a course in Computer Science? The book promotes non-science, nonsense, superstition, mythology, and false views of what computers can and cannot do.

There are two parts. Part I is a jolly and light history and survey of computing. There is nothing wrong here, and even a few Fortran programs that may work, but nothing that would challenge a bright undergraduate to think.

Part II is devoted to long quotations from literature (Rossum's Universal Robots, The Desk Set, The Billion Dollar Brain, Giles Goat-Boy, and others). This is science fiction, and undoubtedly fascinating, but terribly out of place. It will appeal strongly to the worst of computing students; namely, those devoted to dreaming of robots, beating the stock market, sure-fire gambling systems, and ESP. And these students need no encouragement; they need guidance.

The book might have a place as a reference, so that the freshman who wants to reinvent robots can be shown that Capek beat him to it in 1920. But how does this advance understanding of the computing art?

--FJG

KENBAK-1 COMPUTER

One student was shocked when told that the little blue box was a computer. The characteristics of the KENBAK-1 computer are a surprise, especially if you have heard the "giant-brain" line of thought. Even though you may be past the point of thinking of computers as "grey-matter", you probably will not be prepared for the fourteen pound briefcase size KENBAK-1 computer. After matching wits with it in a game of heads or tails in which the computer predicts your choice, you may wonder whether it isn't at least a "small-brain". After all, if the computer does win and you do have a brain, then must not the computer with its programs have some intelligence?

One of the reasons that the KENBAK-1 computer came into existence was to give students, and people in general, a chance to learn something about computers and programming. Though many computers exist today, they tend to be remote and inaccessible. The KENBAK-1 computer is meant to be accessible and available for use, for study, or for play. For this, a computer shouldn't have a high price tag. The real shocker in the KENBAK-1 computer is that it costs less than \$1,000.

On showing the computer to people, a lot of them assume it is a calculator or a terminal. Actually, it is a complete self-contained computer which operates internally in the same way as large computers do. Though physically small, it has many advanced technical features. What makes the KENBAK-1 computer possible is its slower speed (though fast enough for its purposes), its smaller memory, and its simple and reliable approach to input and output. To enter numbers into the machine, you press keys on the front panel. Lights allow you to read numbers.

Perhaps the best way to describe the KENBAK-1 computer is to say that it is for the study of concepts and not for problem solving, though this is not entirely true. Writing a program to add two 10 digit numbers is an excellent study of indirect and indexed addressing and subroutines. Writing a program to sort a list of numbers is instructive. But if you wanted to balance your checkbook, using the KENBAK-1 computer would be the hard way. What is more fun, and just as instructive, is to write programs to play "games". Games have the same data organization problems, decision making and logical analysis that arithmetic problems do. The interaction with the computer stimulates the user's interest and re-inforces his understanding of the concepts.

Most KENBAK-1 computers are in formal education. The low price appeals to budget limited schools. It also appeals to schools who are aware of the congestion which develops around one computer or one terminal. A Commission on Education Task Force stated that it is vital for students to program and to run their own programs directly on computers. This hands-on programming experience is an important teaching aid. The intimate relationship that develops between a student and a machine is highly motivational. The immediate feedback forces the student to develop proper work habits and it quickly indicates those concepts and ideas that he does not fully understand.

Zigzag

In the two columns of numbers below, the numbers on the right are the cube roots of those on the left. A zigzag pattern is followed down the columns, as shown by the dotted line. On the even numbered lines, a cube root is taken to go from left to right; on the odd numbered lines, cubing is used to go from right to left. To proceed down the column, the most recent difference pattern is extended. Thus, to move from line 4 to line 5 (right hand column), the number 1.906 is calculated by:

$$(1.713 - 1.520) + 1.713 = 2(1.713) - 1.520.$$

and similarly on the left in proceeding from an odd to an even numbered line. For the table given here, all calculations are held to 4 significant digits.

①	1.000	1.000	←
②	2.000	1.260	→
③	3.512	1.520	→
④	5.024	1.713	→
⑤	6.924	1.906	→
⑥	8.824	2.066	
⑦	11.03	2.226	
⑧	13.24	2.366	
⑨	15.74	2.506	

The problem is: what will be on the 100th line? The answer would be simple, and easy to calculate, except that it is a function of the precision involved. Thus, the numbers on the 100th line will be significantly different (not just more precise) if all calculations are held to 5S, or 6S, or 12S. Apparently there will be a unique result for each level of precision used in the calculations.

The cube roots required in ZIGZAG may be calculated by logarithms:

$$x = \exp((1/3)(\ln N)) \quad \textcircled{1}$$

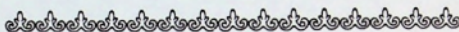
or by iterating with the Newton-Raphson method:

$$x_{n+1} = \frac{2x_n + N}{3x_n^2} \quad \textcircled{2}$$

or by iterating with one of the following two formulas, which capitalize on existing capability to extract square roots:

$$x_{n+1} = (1/3) \left[2\sqrt{\frac{N}{x_n}} + x_n \right]$$

$$x_{n+1} = (1/3) \left[4\sqrt[4]{Nx_n} - x_n \right]$$



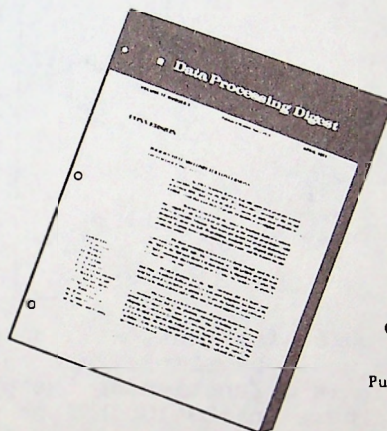
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